BRST Invariance and Renormalisability of the SU(2)×U(1) Electroweak Theory with Massive W Z Bosons

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Abstract

Since the SU(n) gauge theory with massive gauge bosons has been proven to be renormalisable we reinvestigate the renormalisability of the $\mathrm{SU}_L(2) \times \mathrm{U}_Y(1)$ electroweak theory with massive W Z bosons. We expound that with the constraint conditions caused by the W Z mass term and the additional condition chosen by us we can performed the quantization and construct the ghost action in a way similar to that used for the massive SU(n) theory. We also show that when the $\delta-$ functions appearing in the path integral of the Green functions and representing the constraint conditions are rewritten as Fourier integrals with Lagrange multipliers λ_a and λ_y, the BRST invariance is kept in the total effective action consisting of the Lagrange multipliers, ghost fields and the original fields. Furthermore, by comparing with the massless theory and with the massive SU(n) theory we find the general form of the divergent part of the generating functional for the regular vertex functions and prove the renormalisability of the theory. It is also clarified that the renormalisability of the theory with the W Z mass term is ensured by that of the massless theory and the massive SU(n) theory.

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I. Introduction

Although the negative answer to the problem of renormalisability of a SU(n) theory with massive gauge bosons is widely known, such theories continue to be studied (see for example Refs. [1-8]). However, since the negative answer had not been voted down, it was naturally difficult to investigate the possibility of directly adding a mass term to the $\mathrm{SU}_L(2) \times \mathrm{U}_Y(1)$ theory. Recently, the renormalisability of the massive SU(n) gauge theory has been proven [1,2]. Therefore we will reinvestigate the $\mathrm{SU}_L(2) \times \mathrm{U}_Y(1)$ theory of S.L.Glashow [9] with the mass term of the W Z fields. The study of the theory including the mass term of the matter fields as well w1ll be reported in Ref. [10].

In order to make appropriate the mass ratio, the W Z mass term must contain a product of the $\mathrm{SU}_L(2)$ and $\mathrm{U}_Y(1)$ fields and thus cause constraint conditions containing products of such fields. Next, such a mass term is invariant under an infinitesimal gauge transformation with $\delta\theta_1$ and $\delta\theta_2$ equal to zero and $\delta\theta_3$ equal to $\delta\theta_y$, where θ_a and θ_1 are the parameters of the gauge group. Therefore an additional constraint condition should be properly chosen. We will expound that with the constraint conditions caused by the W Z mass term and the additional condition chosen by us we can performed the quantization and construct the ghost action in a way similar to that used for the massive SU(n) theory [1]. We will also show that when the $\delta-$ functions appearing in the path integral of the Green functions and representing the constraint conditions are rewritten as Fourier integrals with Lagrange multipliers λ_a and λ_y, the BRST invariance is kept in the total effective action consisting of the Lagrange multipliers, ghost fields and the original fields.

As the constraint conditions contain the products of the $\mathrm{SU}_L(2)$ and $\mathrm{U}_Y(1)$ fields, the divergent part of the generating functional Γ for the regular vertex functions is dependent on the classical fields of the Lagrange multipliers λ_a and λ_y when the generating functional for the Green functions contains the sources of these Lagrange multipliers. The problem of whether such a generalized form of the theory is renormalisable becomes complicated. However, we are not interested in using the Green functions involving λ_a or λ_y. Thus we can avoid introducing the sources of these Lagrange multipliers to the generating functional for the Green functions. An equivalent and convenient procedure is to derive the Slavnov–Taylor identities and the additional identities for Γ with the help of the generalized form of the theory and then let vanish the functional derivatives of Γ with respect to the classical fields of these Lagrange multipliers. In this way the divergent part of Γ will be shown to satisfy the same equations appearing in the massless theory. Furthermore, by comparing with the massless theory and with the massive SU(n) theory we will be able to find the general form of the divergent part of Γ and prove the renormalisability of the theory. Meanwhile it will be clarified that the renormalisability of the theory with the W Z mass term is ensured by that of the massless theory and the massive SU(n) theory.

In section 2 we will find the constraint conditions caused by the W Z mass term. The additional constraint condition will also be chosen. The method of quantization will be explained in section 3. Setion 4 is devoted to prove the renormalisability of the theory. Concluding remarks will be given in the final section.

II. Original and Additional Constraint Conditions

For the sake of comvenience we assume in the present work that the matter fields consist only of the electron and electron-neutrino fields and are often denoted by $\psi(x)$ and $\overline{\psi}(x)$. The former stands for the purely left-handed neutrino field ν_L, the left- and right-handed parts of the electron field namely e_L, e_R, and the latter stands for $\overline{\nu}_L$, $\overline{e}_L$ and $\overline{e}_R$. Next let $W_{a\mu}(x)$, $W_{y\mu}(x)$ be the $\mathrm{SU}_L(2)$ and $\mathrm{U}_Y(1)$ gauge fields and g, g_1 be the coupling constants. Thus the W Z mass term in the Lagrangian is

$$\mathcal{L}_{WM} = \frac{1}{2}M^2 W_{a\mu}W_a^{\mu} + \frac{1}{2}M^2\Big(\frac{g_1}{g}\Big)^2 W_{y\mu}W_y^{\mu} - M^2\Big(\frac{g_1}{g}\Big)W_{3\mu}W_y^{\mu}\,, \tag{2.1}$$

or

$$\mathcal{L}_{WM} = \frac{1}{2}M^2 W_{1\mu}(x)W_1^{\mu}(x) + \frac{1}{2}M^2 W_{2\mu}(x)W_2^{\mu}(x) + \frac{1}{2}M_z^2 Z_{\mu}(x)Z^{\mu}(x)\,,$$

where M_z^2 stands for $g^{-2}(g^2+g_1^2)M^2$, and $Z_\mu(x)$, $A_\mu(x)$ are the field functions of Z boson and photon, namely

$$Z_\mu = \frac{1}{\sqrt{(g^2+g_1^2)}}(gW_{3\mu} - g_1 W_{y\mu})\,, \tag{2.2}$$

$$A_\mu = \frac{1}{\sqrt{(g^2+g_1^2)}}\varepsilon(g_1 W_{3\mu} + gW_{y\mu})\,, \tag{2.3}$$

where ε is 1 or -1.

The original Lagrangian of the $\mathrm{SU}_L(2)\times\mathrm{U}_Y(1)$ electroweak theory with the mass term $\mathcal{L}_{WM}$ is

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_{\psi W} + \mathcal{L}_{WM} + \mathcal{L}_{WL} + \mathcal{L}_{WY}\,, \tag{2.4}$$

where $\mathcal{L}_\psi$ describe the pure matter fields, $\mathcal{L}_{\psi W}$ is the coupling term between the matter and gauge fields and

$$\mathcal{L}_{WL} = -\frac{1}{4} F_{a\mu\nu} F_a^{\mu\nu} \,, \tag{2.5}$$

$$\mathcal{L}_{WY} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} \,, \tag{2.6}$$

with

$$F_{a\mu\nu} = \partial_\mu W_{a\nu} - \partial_\nu W_{a\mu} - g C_{abc} W_{b\mu} W_{c\nu} \,, \tag{2.7}$$

$$B_{\mu\nu} = \partial_\mu W_{y\nu} - \partial_\nu W_{y\mu} \,. \tag{2.8}$$

C_{abc} stands for the structure constants of $\mathrm{SU}_L(2)$ with C_{123} equal to 1.

Denote by $\theta_a(x), \theta_y(x)$ the parameters of the gauge group. Thus, under an infinitesimal gauge transformation, the fields W_a^μ, W_y^μ, ψ and $\overline{\psi}$ transform as

$$\begin{aligned}
&\delta W_a^\mu(x) = -\frac{1}{g}\partial^\mu \delta\theta_a(x) - C_{abc} W_c^\mu(x) \delta\theta_b(x) \,, \\
&\delta W_y^\mu(x) = -\frac{1}{g_1}\partial^\mu \delta\theta_y(x) \,, \\
&\delta \nu_L(x) = \frac{i}{2}\delta\theta_1(x) e_L(x) + \frac{1}{2}\delta\theta_2(x) e_L(x) + \frac{i}{2}\delta\theta_3(x)\nu_L(x) - \frac{i}{2}\delta\theta_y(x)\nu_L(x) \,, \\
&\delta e_L(x) = \frac{i}{2}\delta\theta_1(x)\nu_L(x) - \frac{1}{2}\delta\theta_2(x)\nu_L(x) - \frac{i}{2}\delta\theta_3(x) e_L(x) - \frac{i}{2}\delta\theta_y(x) e_L(x) \,, \\
&\delta e_R(x) = -i\delta\theta_y(x) e_R(x) \,, \\
&\delta \overline{\nu}_L(x) = -\frac{i}{2}\delta\theta_1(x)\overline{e}_L(x) + \frac{1}{2}\delta\theta_2(x)\overline{e}_L(x) - \frac{i}{2}\delta\theta_3(x)\overline{\nu}_L(x) + \frac{i}{2}\delta\theta_y(x)\overline{\nu}_L(x) \,, \\
&\delta \overline{e}_L(x) = -\frac{i}{2}\delta\theta_1(x)\overline{\nu}_L(x) - \frac{1}{2}\delta\theta_2(x)\overline{\nu}_L(x) + \frac{i}{2}\delta\theta_3(x)\overline{e}_L(x) + \frac{i}{2}\delta\theta_y(x)\overline{e}_L(x) \,, \\
&\delta \overline{e}_R(x) = i\delta\theta_y(x)\overline{e}_R(x) \,.
\end{aligned}$$

Therefore the action transforms as

$$\begin{aligned}
\delta \int d^4x \mathcal{L}(x) &= \delta \int d^4x \mathcal{L}_{WM}(x) \\
&= \int d^4x \Big\{ \Big(\frac{M^2}{g}\partial_\mu W_1^\mu(x) + \frac{M^2}{g} g_1 W_{2\mu}(x) W_y^\mu(x) \Big) \delta\theta_1 \\
&\quad + \Big(\frac{M^2}{g}\partial_\mu W_2^\mu(x) - \frac{M^2}{g} g_1 W_{1\mu}(x) W_y^\mu(x) \Big) \delta\theta_2 \\
&\quad + \Big(\frac{M^2}{g}\partial_\mu W_3^\mu(x) - \frac{M^2}{g^2} g_1 \partial_\mu W_y^\mu(x) \Big) (\delta\theta_3 - \delta\theta_y) \Big\} \,.
\end{aligned} \tag{2.9}$$

Since the classical equations of motion make the action invariant under an arbitrary infinitesimal transformation of the field functions, they certainly make the W Z mass term invariant under an arbitrary infinitesimal gauge transformation. This means that when M is not equal to zero, the classical equations of motion leads to the following constraint conditions

$$\frac{M^2}{g}\partial_\mu W_1^\mu(x)+\frac{M^2}{g}g_1W_{2\mu}(x)W_y^\mu(x)=0\,, \tag{2.10}$$

$$\frac{M^2}{g}\partial_\mu W_2^\mu(x)-\frac{M^2}{g}g_1W_{1\mu}(x)W_y^\mu(x)=0\,, \tag{2.11}$$

$$\frac{M^2}{g}\partial_\mu W_3^\mu(x)-\frac{M^2}{g^2}g_1\partial_\mu W_y^\mu(x)=0\,. \tag{2.12}$$

These are the original constraint conditions. As it can be seen from (2.9) that the W Z mass term is invariant under an infinitesimal gauge transformation with $\delta\theta_1$ and $\delta\theta_2$ equal to zero and $\delta\theta_3$ equal to $\delta\theta_y$. For this reason, $\partial_\mu W_3^\mu$ and $\partial_\mu W_y^\mu$ appear in one constraint. We now choose an additional condition and replace (2.12) with

$$\frac{M^2}{g}\partial_\mu W_3^\mu(x)+\frac{M^2}{g}g_1W_{3\mu}(x)W_y^\mu(x)=0\,, \tag{2.13}$$

$$\partial_\mu W_y^\mu(x)+gW_{3\mu}(x)W_y^\mu(x)=0\,. \tag{2.14}$$

III. Quantization and BRST Invariance

Write (2.10), (2.11) and (2.13),(2.14) as

$$\Phi_a(x)=0\,,\qquad \Phi_y(x)=0\,, \tag{3.1}$$

with

$$\Phi_1(x)=\partial_\mu W_1^\mu(x)+g_1W_{2\mu}(x)W_y^\mu(x)\,, \tag{3.2}$$

$$\Phi_2(x)=\partial_\mu W_2^\mu(x)-g_1W_{1\mu}(x)W_y^\mu(x)\,, \tag{3.3}$$

$$\Phi_3(x)=\partial_\mu W_3^\mu(x)+g_1W_{3\mu}(x)W_y^\mu(x)\,, \tag{3.4}$$

$$\Phi_y(x)=\partial_\mu W_y^\mu(x)+gW_{3\mu}(x)W_y^\mu(x)\,. \tag{3.5}$$

Taking the constraint conditions (3.1) into account one should write the path integral of the Green functions inolving only the original fields as

$$\frac{1}{N_0}\int\mathcal{D}[\mathcal{W},\overline{\psi},\psi]\Delta[\mathcal{W},\overline{\psi},\psi]\prod_{a',x'}\delta\left(\Phi_{a'}(x')\right)\delta\left(\Phi_y(x')\right)W_{a\mu}(x)W_{b\nu}(y)\cdots\exp\{\mathrm{i}I\}\,, \tag{3.6}$$

where

$$
\begin{aligned}
I &= \int d^4x\mathcal{L}(x)\,, \\
N_0 &= \int \mathcal{D}[\mathcal{W},\overline{\psi},\psi]\Delta[\mathcal{W},\overline{\psi},\psi]\prod_{a',x'}\delta\left(\Phi_{a'}(x')\right)\delta\left(\Phi_y(x')\right)\exp\{\mathrm{i}I\}\,.
\end{aligned}
$$

The weight factor $\Delta[\mathcal{W},\overline{\psi},\psi]$ is to be determined. Since only the field functions which satisfy the constraint conditions can play roles in the integral (3.6), the value of the Lagrangian can be changed for the field functions which do not satisfy these conditions. In view of the fact that the conditions (3.1) make the action invariant with respect to the infinitesimal gauge trasformation, we now imagine to replace the mass term $\mathcal{L}_{WM}$ in (3.6) with a gauge invariant mass term which is equal to $\mathcal{L}_{WM}$ when the conditions (3.1) are satisfied. Thus, analogous to the case in the Fadeev–Popov method [1,11-16], $\Delta[\mathcal{W},\overline{\psi},\psi]$ should be gauge invariant and make the following equation valid for an arbitrary gauge invariant quantity $\mathcal{O}(\mathcal{W},\,\overline{\psi},\,\psi)$

$$
\begin{aligned}
&\int \mathcal{D}[\mathcal{W},\,\overline{\psi},\,\psi]\Delta[\mathcal{W},\overline{\psi},\psi]\prod_{a',x'}\delta\left(\Phi_{a'}(x')\right)\delta\left(\Phi_y(x')\right)\mathcal{O}(\mathcal{W},\overline{\psi},\psi)\exp\{\mathrm{i}\widetilde{I}\} \\
&\qquad\propto \int \mathcal{D}[\mathcal{W},\,\overline{\psi},\,\psi]\mathcal{O}(\mathcal{W},\overline{\psi},\psi)\exp\{\mathrm{i}\widetilde{I}\}\,.
\end{aligned}
$$

where $\widetilde{I}$ is a gauge invariant action constructed by replacing $\mathcal{L}_{WM}$ with the imagined mass term. This means that the weight factor $\Delta[\mathcal{W},\overline{\psi},\psi]$ can be determined according to the Fadeev–Popov equation of the following form

$$
\Delta[\mathcal{W},\overline{\psi},\psi]\int\prod_{z}d\Omega(z)\prod_{\sigma,x}\delta\Big(\Phi_\sigma^\Omega(x)\Big) = 1\,. \tag{3.7}
$$

where σ stands for $1,2,3,y$, $\Phi_\sigma^\Omega(x)$ is the result of acting on $\Phi_\sigma(x)$ with a gauge transformation having the parameters of the element $\Omega(x)$ of the gauge group, $d\Omega(z)$ is the volume element of the group integral. It follows that with the F–P ghost fields $C_a(x)$, $C_y(x)$, $\overline{C}_a(x)$, $\overline{C}_y(x)$ as new variables, one can express the ghost Lagrangian as

$$
\mathcal{L}^{(C)}(x) = \overline{C}_a(x)\Delta\Phi_a(x) + \overline{C}_y(x)\Delta\Phi_y(x)\,, \tag{3.8}
$$

where $\Delta\Phi_a(x)$, $\Delta\Phi_y(x)$ are defined by the BRST transformtion of $\Phi_a(x)$ and $\Phi_y(x)$ so that

$$
\delta_B\Phi_a(x) = \delta\zeta\Delta\Phi_a(x)\,,\qquad \delta_B\Phi_y(x) = \delta\zeta\Delta\Phi_y(x)\,, \tag{3.9}
$$

where $\delta\zeta$ is an infinitesimal fermionic parameter independent of x. The BRST transformation of the gauge fields or matter fields is nothing but the infinitesimal gauge transformation with $\delta\theta_a$ and $\delta\theta_y$ equal to $-g\delta\zeta C_a$ and $-g_1\delta\zeta C_y$ respectively. Namely

$$\delta_B W_a^\mu(x) = \delta\zeta \Delta W_a^\mu(x) = \delta\zeta D_{ab}^\mu C_b(x)\,, \tag{3.10}$$

$$\delta_B W_y^\mu(x) = \delta\zeta \Delta W_y^\mu(x) = \delta\zeta \partial^\mu C_y(x)\,, \tag{3.11}$$

$$\delta_B \psi(x) = \delta\zeta \Delta\psi(x)\,, \qquad \delta_B \overline{\psi}(x) = \delta\zeta \Delta\overline{\psi}(x)\,, \tag{3.12}$$

where

$$\begin{aligned}
&D_{ab}^\mu(x) = \delta_{ab}\partial^\mu + g f_{abc} A_c^\mu(x)\,,\\
&\Delta\nu_L(x) = -\frac{i}{2} g C_1(x) e_L(x) - \frac{1}{2} g C_2(x) e_L(x) - \frac{i}{2} g C_3(x)\nu_L(x) + \frac{i}{2} g_1 C_y(x)\nu_L(x)\,,\\
&\Delta e_L(x) = -\frac{i}{2} g C_1(x)\nu_L(x) + \frac{1}{2} g C_2(x)\nu_L(x) + \frac{i}{2} g C_3(x) e_L(x) + \frac{i}{2} g_1 C_y(x) e_L(x)\,,\\
&\Delta e_R(x) = i g_1 C_y(x) e_R(x)\,,\\
&\Delta\overline{\nu}_L(x) = \frac{i}{2} g C_1(x)\overline{e}_L(x) - \frac{1}{2} g C_2(x)\overline{e}_L(x) + \frac{i}{2} g C_3(x)\overline{\nu}_L(x) - \frac{i}{2} g_1 C_y(x)\overline{\nu}_L(x)\,,\\
&\Delta\overline{e}_L(x) = \frac{i}{2} g C_1(x)\overline{\nu}_L(x) + \frac{1}{2} g C_2(x)\overline{\nu}_L(x) - \frac{i}{2} g C_3(x)\overline{e}_L(x) - \frac{i}{2} g_1 C_y(x)\overline{e}_L(x)\,,\\
&\Delta\overline{e}_R(x) = -i g_1 C_y(x)\overline{e}_R(x)\,.
\end{aligned}$$

$C_a(x)$ and $C_y(x)$ are also transformed as usual

$$\begin{aligned}
&\delta_B C_a(x) = \delta\zeta \Delta C_a(x) = \delta\zeta \frac{g}{2} C_{abc} C_b(x) C_c(x)\,,\\
&\delta_B C_y(x) = 0\,.
\end{aligned}$$

Now we can write $\Delta\Phi_a(x)$, $\Delta\Phi_y(x)$ as

$$\Delta\Phi_1 = \partial_\mu \Delta W_1^\mu(x) + g_1 \Delta W_2^\mu(x) W_{y\mu}(x) + g_1 W_{2\mu}(x)\Delta W_y^\mu(x)\,, \tag{3.13}$$

$$\Delta\Phi_2 = \partial_\mu \Delta W_2^\mu(x) - g_1 \Delta W_1^\mu(x) W_{y\mu}(x) - g_1 W_{1\mu}(x)\Delta W_y^\mu(x)\,, \tag{3.14}$$

$$\Delta\Phi_3 = \partial_\mu \Delta W_3^\mu(x) + g_1 \Delta W_3^\mu(x) W_{y\mu}(x) + g_1 W_{3\mu}(x)\Delta W_y^\mu(x)\,, \tag{3.15}$$

$$\Delta\Phi_y = \partial_\mu \Delta W_y^\mu(x) + g \Delta W_3^\mu(x) W_{y\mu}(x) + g W_{3\mu}(x)\Delta W_y^\mu(x)\,, \tag{3.16}$$

Since ΔW_a^μ, ΔW_y^μ, $\Delta\psi(x)$, $\Delta\overline{\psi}(x)$ and $\Delta C_a(x)$ are BRST invariant, it is easy to see that $\Delta\Phi_a(x)$ and $\Delta\Phi_y(x)$ are also BRST invariant.

One can further generalized the theory by regarding as new variables the Lagrange multipliers $\lambda_a(x)$ and $\lambda_y(x)$ associated with the constraint conditions. Thus the total effective Lagrangian and action consist of these Lagrange multipliers, ghosts and the original variables, namely

$$\mathcal{L}_{\rm eff}(x) = \mathcal{L}(x) + \mathcal{L}^{(C)}(x) + \lambda_a(x)\Phi_a(x) + \lambda_y(x)\Phi_y(x)\,, \tag{3.17}$$

$$I_{\rm eff}=\int d^4x\mathcal{L}_{\rm eff}(x)\,. \tag{3.18}$$

Correspondingly, the path integral of the generating functional for the Green functions is

$$\mathcal{Z}[\overline{\eta},\eta,\overline{\chi},\chi,J,j]=\frac{1}{N_\lambda}\int\mathcal{D}[\overline{\psi},\psi,\mathcal{W},\overline{C},C,\lambda]{\rm exp}\Big\{{\rm i}\big(I_{\rm eff}+I_s\big)\Big\}, \tag{3.19}$$

where N_λ is a constant, I_s is the source term in the action. They are defined by

$$\begin{aligned}N_\lambda=&\int\mathcal{D}[\overline{\psi},\psi,\mathcal{W},\overline{C},C,\lambda]{\rm exp}\Big\{{\rm i}I_{\rm eff}\Big\}\,,\\ I_s=&\int d^4x\Big\{\overline{\eta}(x)\psi(x)+\overline{\psi}(x)\eta(x)+\overline{\chi}_a(x)C_a(x)+\overline{C}_a(x)\chi_a(x)+\overline{\chi}_y(x)C_y(x)\\ &+\overline{C}_y(x)\chi_y(x)+J_a^\mu(x)W_{a\mu}(x)+J_y^\mu(x)W_{y\mu}(x)+j_a(x)\lambda_a(x)+J_y(x)\lambda_y(x)\Big\}\,,\end{aligned} \tag{3.20}$$

where $\overline{\eta}(x),\eta(x)\cdots$ stand for the sources. In particular, $j_a(x)$, $j_y(x)$ are the sources of $\lambda_a(x)$, $\lambda_y(x)$, respectively.

We now check the BRST invariance of the effective action I_{eff} defined by (3.17) and (3.18). With $\overline{C}_a(x)$, $\overline{C}_y(x)$ transforming as

$$\delta_B\overline{C}_a(x)=-\delta\zeta\lambda_a(x)\,,\qquad \delta_B\overline{C}_y(x)=-\delta\zeta\lambda_y(x)\,,$$

and noticing the invariance of $\Delta\Phi_a,\Delta\Phi_y$, one has

$$\delta_B\int d^4x\mathcal{L}^{(C)}(x)=\int d^4x\Big\{-\lambda_a(x)\delta_B\Phi_a(x)-\lambda_y(x)\delta_B\Phi_y(x)\Big\}\,.$$

Therefore

$$\delta_BI_{eff}=\delta_BI_{WM}+\int d^4x\Big\{\big(\delta_B\lambda_a(x)\big)\Phi_a(x)+\big(\delta_B\lambda_y(x)\big)\Phi_y(x)\Big\}\,.$$

From this and the expression of δ_BI_{WM}, it can be shown that the effective action is invariant, when the transformation of $\lambda_a(x)$ and $\lambda_y(x)$ are defined as

$$\begin{aligned}\delta_B\lambda_1(x)&=\delta\zeta M^2C_1(x)\,,\\ \delta_B\lambda_2(x)&=\delta\zeta M^2C_2(x)\,,\\ \delta_B\lambda_3(x)&=\delta\zeta M^2C_3(x)-\delta\zeta\frac{g_1}{g}M^2C_y(x)\,,\\ \delta_B\lambda_y(x)&=\delta\zeta\frac{g_1^2}{g^2}M^2C_y(x)-\delta\zeta\frac{g_1}{g}M^2C_3(x)\,.\end{aligned}$$

IV. Renormalisability

Let $W_{a\mu}(x), W_{y\mu}(x),\ C_a(x), C_y(x), \cdots$ stand for the renormalized field founctions, g, g_1 and M be renormalized parameters. By introducing the source terms of the composite field functions ΔW_a^μ, ΔW_y^μ, $\Delta C_a(x)$, $\Delta\psi(x)$, $\Delta\overline{\psi}(x)$ and the sources $K_\mu^a(x)$, $K_\mu^y(x)$, $L_a(x)$, $n_\alpha(x)$, $l_\alpha(x)$, $p_\alpha(x)$, $n'_\alpha(x)$, $l'_\alpha(x)$ and $p'_\alpha(x)$, the effective Lagrangian without counterterm becomes

$$\begin{aligned}\mathcal{L}_{eff}^{[0]}(x) \;=\;& \lambda_a(x)\Phi_a(x)+\lambda_y(x)\Phi_y(x)+\mathcal{L}_{WL}(x)+\mathcal{L}_{WY}(x)\\ &+\mathcal{L}_{WM}(x)+\mathcal{L}^{(C)}(x)+\mathcal{L}_\psi(x)+\mathcal{L}_{\psi W}(x)\\ &+K_\mu^a(x)\Delta W_a^\mu(x)+K_\mu^y(x)\Delta W_y^\mu(x)+L_a(x)\Delta C_a(x)\\ &+n_\alpha(x)\Delta\nu_{L\alpha}(x)+l_\alpha(x)\Delta e_{L\alpha}(x)+p_\alpha(x)\Delta e_{R\alpha}(x)\\ &+n'_\alpha(x)\Delta\overline{\nu}_{L\alpha}(x)+l'_\alpha(x)\Delta\overline{e}_{L\alpha}(x)+p'_\alpha(x)\Delta\overline{e}_{R\alpha}(x)\,. \end{aligned} \tag{4.1}$$

The complete effective Lagrangian is the sum of $\mathcal{L}_{eff}^{[0]}$ and the counterterm $\mathcal{L}_{count}$

$$\mathcal{L}_{\rm eff}=\mathcal{L}_{\rm eff}^{[0]}+\mathcal{L}_{count}\,. \tag{4.2}$$

With (4.1), the generating functional for Green functions is defined as

$$\mathcal{Z}^{[0]}[\overline{\eta},\eta,\overline{\chi},\chi,J,j,K,L,n,l,p,n',l',p']=\frac{1}{N}\int\mathcal{D}[\overline{\psi},\psi,\mathcal{W},\overline{C},C,\lambda]\exp\Big\{{\rm i}\big(I_{eff}^{[0]}+I_s\big)\Big\}\,, \tag{4.3}$$

$I_{eff}^{[0]}$ is the effective action $\int d^4x\mathcal{L}_{eff}^{[0]}(x)$, N is a constant to make $\mathcal{Z}^{[0]}$ equal to 1 in the absence of

$$\begin{aligned}I_s=\int d^4x\Big\{&\overline{\eta}(x)\psi(x)+\overline{\psi}(x)\eta(x)+\overline{\chi}_a(x)C_a(x)+\overline{C}_a(x)\chi_a(x)+\overline{\chi}_y(x)C_y(x)\\ &+\overline{C}_y(x)\chi_y(x)+J_a^\mu(x)W_{a\mu}(x)+J_y^\mu(x)W_{y\mu}(x)+j_a(x)\lambda_a(x)+j_y(x)\lambda_y(x)\Big\}\,,\end{aligned}$$

where $\overline{\eta}\,\psi$ and $\overline{\psi}\,\eta$ stand for

$$\begin{aligned}\overline{\eta}\,\psi&=\overline{\eta}_\alpha^{(\nu)}\,\nu_{L\alpha}+\overline{\eta}_\alpha^{(l)}\,e_{L\alpha}+\overline{\eta}_\alpha^{(r)}\,e_{R\alpha}\,,\\ \overline{\psi}\,\eta&=\overline{\nu}_{L\alpha}\,\eta_\alpha^{(\nu)}+\overline{e}_{L\alpha}\,\eta_\alpha^{(l)}+\overline{e}_{R\alpha}\,\eta_\alpha^{(r)}\,.\end{aligned}$$

Denoting by $\mathcal{W}^{[0]}$ and $\Gamma^{[0]}$ the generating functionals for connected Green functions and regular vertex functions respectively, one has

$$\mathcal{Z}^{[0]}=\exp\Big\{{\rm i}\mathcal{W}^{[0]}[\overline{\eta},\eta,\overline{\chi},\chi,J,j,K,L,n,l,p,n',l',p']\Big\}\,, \tag{4.4}$$

$$
\begin{aligned}
&\Gamma^{[0]}[\widetilde{\psi},\widetilde{\overline{\psi}},\widetilde{W},\widetilde{\overline{C}},\widetilde{C},\widetilde{\lambda},K,L,n,l,p,n',l',p'] \\
&\quad = \mathcal{W}^{[0]} - \int d^4x \Big[J_a^\mu \widetilde{W}_{a\mu} + J_y^\mu \widetilde{W}_{y\mu} + j_a \widetilde{\lambda}_a + j_y \widetilde{\lambda}_y + \overline{\chi}_a \widetilde{C}_a + \widetilde{\overline{C}}_a \chi_a + \overline{\chi}_y \widetilde{C}_y \\
&\qquad + \widetilde{\overline{C}}_y \chi_y + \overline{\eta}^{(\nu)} \widetilde{\nu}_L + \overline{\eta}^{(l)} \widetilde{e}_L + \overline{\eta}^{(r)} \widetilde{e}_R + \widetilde{\overline{\nu}}_L \eta^{(\nu)} + \widetilde{\overline{e}}_L \eta^{(l)} + \widetilde{\overline{e}}_R \eta^{(r)} \Big] , \qquad (4.5)
\end{aligned}
$$

where $\widetilde{W}_{a\mu}$, $\widetilde{\nu}_L$, $\cdots$ are the so-called classical fields defined by

$$
\begin{aligned}
&\widetilde{W}_{a\mu}(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta J_a^\mu(x)} , \qquad \widetilde{\lambda}_a(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta j_a(x)} , \qquad \widetilde{C}_a(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta \overline{\chi}_a(x)} , \\
&\widetilde{\overline{C}}_a(x) = -\frac{\delta \mathcal{W}^{[0]}}{\delta \chi_a(x)} , \qquad \widetilde{W}_{y\mu}(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta J_y^\mu(x)} , \qquad \widetilde{\lambda}_y = \frac{\delta \mathcal{W}^{[0]}}{\delta j_y(x)} , \\
&\widetilde{C}_y(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta \overline{\chi}_y(x)} , \qquad \widetilde{\overline{C}}_y(x) = -\frac{\delta \mathcal{W}^{[0]}}{\delta \chi_y(x)} , \qquad \widetilde{\nu}_{L\alpha}(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta \overline{\eta}_\alpha^{(\nu)}(x)} , \\
&\widetilde{e}_{L\alpha}(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta \overline{\eta}_\alpha^{(l)}(x)} , \qquad \widetilde{e}_{R\alpha}(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta \overline{\eta}_\alpha^{(r)}(x)} , \qquad \widetilde{\overline{\nu}}_{L\alpha}(x) = -\frac{\delta \mathcal{W}^{[0]}}{\delta \eta_\alpha^{(\nu)}(x)} , \\
&\widetilde{\overline{e}}_{L\alpha}(x) = -\frac{\delta \mathcal{W}^{[0]}}{\delta \eta_\alpha^{(l)}(x)} , \qquad \widetilde{\overline{e}}_{R\alpha}(x) = -\frac{\delta \mathcal{W}^{[0]}}{\delta \eta_\alpha^{(r)}(x)} .
\end{aligned}
$$

Therefore

$$
\begin{aligned}
&J_a^\mu(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{W}_{a\mu}(x)} , \qquad j_a(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\lambda}_a(x)} , \qquad \overline{\chi}_a(x) = \frac{\delta \Gamma^{[0]}}{\delta \widetilde{C}_a(x)} , \\
&\chi_a(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\overline{C}}_a(x)} , \qquad J_y^\mu(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{W}_{y\mu}(x)} , \qquad j_y(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\lambda}_y(x)} , \\
&\overline{\chi}_y(x) = \frac{\delta \Gamma^{[0]}}{\delta \widetilde{C}_y(x)} , \qquad \chi_y(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\overline{C}}_y(x)} , \qquad \eta_\alpha^{(\nu)}(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\overline{\nu}}_{L\alpha}(x)} , \\
&\eta_\alpha^{(l)}(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\overline{e}}_{L\alpha}(x)} , \qquad \eta_\alpha^{(r)}(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\overline{e}}_{R\alpha}(x)} , \qquad \overline{\eta}_\alpha^{(\nu)}(x) = \frac{\delta \Gamma^{[0]}}{\delta \widetilde{\nu}_{L\alpha}(x)} , \\
&\overline{\eta}_\alpha^{(l)}(x) = \frac{\delta \Gamma^{[0]}}{\delta \widetilde{e}_{L\alpha}(x)} , \qquad \overline{\eta}_\alpha^{(r)}(x) = \frac{\delta \Gamma^{[0]}}{\delta \widetilde{e}_{R\alpha}(x)} .
\end{aligned}
$$

Besides, for $K_\mu^a, L_a \cdots$, the spectators in the Legendre transtrormation, one has

$$
\begin{aligned}
&\frac{\delta \mathcal{W}^{[0]}}{\delta K_\mu^a(x)} = \frac{\delta \Gamma^{[0]}}{\delta K_\mu^a(x)} , \qquad \frac{\delta \mathcal{W}^{[0]}}{\delta K_\mu^y(x)} = \frac{\delta \Gamma^{[0]}}{\delta K_\mu^y(x)} , \qquad \frac{\delta \mathcal{W}^{[0]}}{\delta L_a(x)} = \frac{\delta \Gamma^{[0]}}{\delta L_a(x)} , \\
&\frac{\delta \mathcal{W}^{[0]}}{\delta n_\alpha(x)} = \frac{\delta \Gamma^{[0]}}{\delta n_\alpha(x)} , \qquad \frac{\delta \mathcal{W}^{[0]}}{\delta l_\alpha(x)} = \frac{\delta \Gamma^{[0]}}{\delta l_\alpha(x)} , \qquad \frac{\delta \mathcal{W}^{[0]}}{\delta p_\alpha(x)} = \frac{\delta \Gamma^{[0]}}{\delta p_\alpha(x)} , \\
&\frac{\delta \mathcal{W}^{[0]}}{\delta n'_\alpha(x)} = \frac{\delta \Gamma^{[0]}}{\delta n'_\alpha(x)} , \qquad \frac{\delta \mathcal{W}^{[0]}}{\delta l'_\alpha(x)} = \frac{\delta \Gamma^{[0]}}{\delta l'_\alpha(x)} , \qquad \frac{\delta \mathcal{W}^{[0]}}{\delta p'_\alpha(x)} = \frac{\delta \Gamma^{[0]}}{\delta p'_\alpha(x)} .
\end{aligned}
$$

In order to find the Slavnov–Taylor identity satisfied by the generating functional for the regular vertex functions, we change the variables in the path integral of $\mathcal{Z}^{[0]}$ as follows

$$
W_a^\mu(x) \to W_a^\mu(x) + \delta\zeta \Delta W_a^\mu(x) , \qquad W_y^\mu(x) \to W_y^\mu(x) + \delta\zeta \Delta W_y^\mu(x) ,
$$

$$
\begin{aligned}
&C_a(x) \to C_a(x) + \delta\zeta\Delta C_a(x)\,, \qquad C_y(x) \to C_y(x)\,, \\
&\overline{C}_a(x) \to \overline{C}_a(x) - \delta\zeta\lambda_a(x)\,, \qquad \overline{C}_y(x) \to \overline{C}_y(x) - \delta\zeta\lambda_y(x)\,, \\
&\psi(x) \to \psi(x) + \delta\zeta\Delta\psi(x)\,, \qquad \overline{\psi}(x) \to \overline{\psi}(x) + \delta\zeta\Delta\overline{\psi}(x)\,, \\
&\lambda_a(x) \to \lambda_a(x)\,, \qquad \lambda_y(x) \to \lambda_y(x)\,.
\end{aligned}
$$

The volume element of the path integral does not change and the changes in I_s and $\mathcal{L}_{WM}$ lead to

$$
\begin{aligned}
\int d^4x \Big\{ & \frac{\delta\Gamma^{[0]}}{\delta K^a_\mu(x)} \frac{\delta\Gamma^{[0]}}{\delta \widetilde{W}^\mu_a(x)} + \frac{\delta\Gamma^{[0]}}{\delta K^y_\mu(x)} \frac{\delta\Gamma^{[0]}}{\delta \widetilde{W}^\mu_y(x)} + \frac{\delta\Gamma^{[0]}}{\delta L_a(x)} \frac{\delta\Gamma^{[0]}}{\delta \widetilde{C}_a(x)} \\
& + \frac{\delta\Gamma^{[0]}}{\delta \widetilde{\nu}_{L\alpha}(x)} \frac{\delta\Gamma^{[0]}}{\delta n_\alpha(x)} + \frac{\delta\Gamma^{[0]}}{\delta \widetilde{e}_{L\alpha}(x)} \frac{\delta\Gamma^{[0]}}{\delta l_\alpha(x)} + \frac{\delta\Gamma^{[0]}}{\delta \widetilde{e}_{R\alpha}(x)} \frac{\delta\Gamma^{[0]}}{\delta p_\alpha(x)} \\
& + \frac{\delta\Gamma^{[0]}}{\delta \widetilde{\overline{\nu}}_{L\alpha}(x)} \frac{\delta\Gamma^{[0]}}{\delta n'_\alpha(x)} + \frac{\delta\Gamma^{[0]}}{\delta \widetilde{\overline{e}}_{L\alpha}(x)} \frac{\delta\Gamma^{[0]}}{\delta l'_\alpha(x)} + \frac{\delta\Gamma^{[0]}}{\delta \widetilde{\overline{e}}_{R\alpha}(x)} \frac{\delta\Gamma^{[0]}}{\delta p'_\alpha(x)} \\
& - \widetilde{\lambda}_a(x) \frac{\delta\Gamma^{[0]}}{\delta \widetilde{\overline{C}}_a(x)} - \widetilde{\lambda}_y(x) + \frac{\delta\Gamma^{[0]}}{\delta \widetilde{\overline{C}}_y(x)} - \langle \Delta\mathcal{L}_{WM}(x)\rangle^{[0]} \Big\} = 0\,,
\end{aligned} \tag{4.6}
$$

where

$$
\langle \Delta\mathcal{L}_{WM}(x)\rangle^{[0]} = \frac{1}{N\mathcal{Z}^{[0]}} \int \mathcal{D}[\overline{\psi}, \psi, \mathcal{W}, \overline{C}, C] \Delta\mathcal{L}_{WM}(x) \mathrm{exp}\Big\{ \mathrm{i}\big(I^{[0]}_{\mathrm{eff}} + I_s\big)\Big\}\,.
$$

With the definition of $\Delta\mathcal{L}_{WM}(x)$

$$
\delta_B \mathcal{L}_{WM}(x) = \delta\zeta\Delta\mathcal{L}_{WM}(x)\,,
$$

one can write

$$
\begin{aligned}
\langle \Delta\mathcal{L}_{WM}(x)\rangle^{[0]} \quad = \quad & M^2 \widetilde{W}_{a\mu}(x) \frac{\delta\Gamma^{[0]}}{\delta K^a_\mu(x)} + M^2 \Big(\frac{g_1}{g}\Big)^2 \widetilde{W}_{y\mu}(x) \frac{\delta\Gamma^{[0]}}{\delta K^y_\mu(x)} \\
& - M^2 \frac{g_1}{g} \widetilde{W}_{y\mu}(x) \frac{\delta\Gamma^{[0]}}{\delta K^3_\mu(x)} - M^2 \frac{g_1}{g} \widetilde{W}_{3\mu}(x) \frac{\delta\Gamma^{[0]}}{\delta K^y_\mu(x)}\,.
\end{aligned}
$$

Next, from the invariance of the path integral of $\mathcal{Z}^{[0]}$ with respect to the translation of the integration variables $\overline{C}_a(x)$, $\overline{C}_y(x)$, $\lambda_a(x)$ and $\lambda_y(x)$, one can get a set of auxiliary identities

$$
\frac{\delta\Gamma^{[0]}}{\delta \widetilde{\overline{C}}_1(x)} - \partial_\mu \frac{\delta\Gamma^{[0]}}{\delta K^1_\mu(x)} - g_1 \widetilde{W}_{y\mu} \frac{\delta\Gamma^{[0]}}{\delta K^2_\mu(x)} - g_1 \widetilde{W}_{2\mu} \frac{\delta\Gamma^{[0]}}{\delta K^y_\mu(x)} = 0\,, \tag{4.7}
$$

$$
\frac{\delta\Gamma^{[0]}}{\delta \widetilde{\overline{C}}_2(x)} - \partial_\mu \frac{\delta\Gamma^{[0]}}{\delta K^2_\mu(x)} + g_1 \widetilde{W}_{y\mu} \frac{\delta\Gamma^{[0]}}{\delta K^1_\mu(x)} + g_1 \widetilde{W}_{1\mu} \frac{\delta\Gamma^{[0]}}{\delta K^y_\mu(x)} = 0\,, \tag{4.8}
$$

$$
\frac{\delta\Gamma^{[0]}}{\delta \widetilde{\overline{C}}_3(x)} - \partial_\mu \frac{\delta\Gamma^{[0]}}{\delta K^3_\mu(x)} - g_1 \widetilde{W}_{y\mu} \frac{\delta\Gamma^{[0]}}{\delta K^3_\mu(x)} - g_1 \widetilde{W}_{3\mu} \frac{\delta\Gamma^{[0]}}{\delta K^y_\mu(x)} = 0\,, \tag{4.9}
$$

$$
\frac{\delta\Gamma^{[0]}}{\delta \widetilde{\overline{C}}_y(x)} - \partial_\mu \frac{\delta\Gamma^{[0]}}{\delta K^y_\mu(x)} - g \widetilde{W}_{y\mu} \frac{\delta\Gamma^{[0]}}{\delta K^3_\mu(x)} - g \widetilde{W}_{3\mu} \frac{\delta\Gamma^{[0]}}{\delta K^y_\mu(x)} = 0\,, \tag{4.10}
$$

and

$$\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\lambda}_a(x)} = \langle\Phi_a(x)\rangle^{[0]}\,, \qquad \frac{\delta\Gamma^{[0]}}{\delta\widetilde{\lambda}_y(x)} = \langle\Phi_y(x)\rangle^{[0]}\,. \tag{4.11}$$

where

$$\langle\Phi_a(x)\rangle^{[0]} = \frac{1}{N\mathcal{Z}^{[0]}}\int\mathcal{D}[\overline{\psi},\psi,\mathcal{W},\overline{C},C,\lambda]\Phi_a(x)\mathrm{exp}\Big\{\mathrm{i}\big(I_{\rm eff}^{[0]}+I_s\big)\Big\}\,, \tag{4.12}$$

$$\langle\Phi_y(x)\rangle^{[0]} = \frac{1}{N\mathcal{Z}^{[0]}}\int\mathcal{D}[\overline{\psi},\psi,\mathcal{W},\overline{C},C,\lambda]\Phi_y(x)\mathrm{exp}\Big\{\mathrm{i}\big(I_{\rm eff}^{[0]}+I_s\big)\Big\}\,. \tag{4.13}$$

Let $\widetilde{\Phi}_a(x)$, $\widetilde{\Phi}_y(x)$, $\widetilde{\mathcal{L}}_{WM}$ be the results obtained from $\Phi_a(x)$, $\Phi_y(x)$, $\mathcal{L}_{WM}$ by replacing the field functions with the classical field functions and define

$$\overline{\Gamma}^{[0]} = \Gamma^{[0]} - \int d^4x\Big\{\widetilde{\lambda}_a(x)\widetilde{\Phi}_a(x)+\widetilde{\lambda}_y(x)\widetilde{\Phi}_y(x)+\widetilde{\mathcal{L}}_{WM}\Big\}\,, \tag{4.14}$$

Thus, from (4.6)–(4.11), one gets

$$\begin{aligned}\int d^4x\Big\{&\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^a(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{W}_a^\mu(x)}+\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^y(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{W}_y^\mu(x)}+\frac{\delta\overline{\Gamma}^{[0]}}{\delta L_a(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{C}_a(x)}\\ &+\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\nu}_{L\alpha}(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta n_\alpha(x)}+\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{e}_{L\alpha}(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta l_\alpha(x)}+\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{e}_{R\alpha}(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta p_\alpha(x)}\\ &+\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{\nu}}_{L\alpha}(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta n'_\alpha(x)}+\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{e}}_{L\alpha}(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta l'_\alpha(x)}+\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{e}}_{R\alpha}(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta p'_\alpha(x)}\Big\}=0\,.\end{aligned} \tag{4.15}$$

and

$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\lambda}_a(x)} = \langle\Phi_a(x)\rangle^{[0]}-\widetilde{\Phi}_a(x)\,, \qquad \frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\lambda}_y(x)} = \langle\Phi_y(x)\rangle^{[0]}-\widetilde{\Phi}_y(x)\,, \tag{4.16}$$

$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{C}}_1(x)}-\partial_\mu\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^1(x)}-g_1\widetilde{W}_{y\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^2(x)}-g_1\widetilde{W}_{2\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^y(x)}=0\,, \tag{4.17}$$

$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{C}}_2(x)}-\partial_\mu\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^2(x)}+g_1\widetilde{W}_{y\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^1(x)}+g_1\widetilde{W}_{1\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^y(x)}=0\,, \tag{4.18}$$

$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{C}}_3(x)}-\partial_\mu\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^3(x)}-g_1\widetilde{W}_{y\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^3(x)}-g_1\widetilde{W}_{3\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^y(x)}=0\,, \tag{4.19}$$

$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{C}}_y(x)}-\partial_\mu\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^y(x)}-g\widetilde{W}_{y\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^3(x)}-g\widetilde{W}_{3\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^y(x)}=0\,. \tag{4.20}$$

As $\Phi_a(x),\Phi_y(x)$ contain the products of the $\mathrm{SU}_L(2)$ and $\mathrm{U}_Y(1)$ fields, (4.16) is complicated unless the generating functional for the Green functions does not contain the sources of the Lagrange multipliers λ_a and λ_y. Actuarely we are not interested in using the Green functions involving λ_a or λ_y. Our intention to use the generalized form of the theory containing the sources of these Lagrange multipliers is to study the Renormalisability of the theory for which such sources are absent from the generating functional for the Green functions and therefore $\langle\Phi_a(x)\rangle^{[0]}$ and $\langle\Phi_y(x)\rangle^{[0]}$ are equal to zero. We now, according to (4.11), let vanish $\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\lambda}_a(x)}$ and $\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\lambda}_a(x)}$ to make $\langle\Phi_a(x)\rangle^{[0]}$ and $\langle\Phi_y(x)\rangle^{[0]}$ equal to zero. This means

$$\widetilde{\Phi}_a(x) = 0\,, \qquad \widetilde{\Phi}_y(x) = 0\,. \tag{4.21}$$

and

$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\lambda}_a(x)} = 0\,, \qquad \frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\lambda}_y(x)} = 0\,, \tag{4.22}$$

In the following we will denote by $\overline{\Gamma}^{[0]}[\psi,\overline{\psi},W,\overline{C},C,\lambda,K,L,n,l,p,n',l',p']$ the functional that is obtained from $\overline{\Gamma}^{[0]}[\widetilde{\psi},\widetilde{\overline{\psi}},\widetilde{W},\widetilde{\overline{C}},\widetilde{C},\widetilde{\lambda},K,\cdots]$ by replacing the classical field functions with the usual field functions. Assume that the dimensional regularization method is used and the Slavnov–Taylor identity and the auxiliary identities are guaranteed. Denote the tree part and one loop part of $\overline{\Gamma}^{[0]}$ by $\overline{\Gamma}^{[0]}_0$ and $\overline{\Gamma}^{[0]}_1$ respectively. $\overline{\Gamma}^{[0]}_0$ is thus the modified action $\overline{I}^{[0]}_{eff}$ obtained from $I^{[0]}_{eff}$ by excluding the mass term and (λ_a,λ_y) terms. From (4.15) and (4.17) – (4.22) one has

$$\Phi_a(x) = 0\,, \qquad \Phi_y(x) = 0\,, \tag{4.23}$$

$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\lambda_a(x)} = 0\,, \qquad \frac{\delta\overline{\Gamma}^{[0]}}{\delta\lambda_y(x)} = 0\,, \tag{4.24}$$

$$\Lambda_{op}\overline{\Gamma}^{[0]}_0 = 0\,,$$

and

$$\overline{\Gamma}^{[0]}_0 * \overline{\Gamma}^{[0]}_1 + \overline{\Gamma}^{[0]}_1 * \overline{\Gamma}^{[0]}_0 = \Lambda_{op}\overline{\Gamma}^{[0]}_1 = 0\,, \tag{4.25}$$

$$\Sigma_a(x)\overline{\Gamma}^{[0]} = 0\,, \qquad \Sigma_y(x)\overline{\Gamma}^{[0]} = 0\,. \tag{4.26}$$

where Λ_{op},$\Sigma_a(x)$ and $\Sigma_y(x)$ are defined by

$$\begin{aligned}\Lambda_{op} = \int d^4x\Big\{ & \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta K^a_\mu(x)}\frac{\delta}{\delta W^\mu_a(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta W^\mu_a(x)}\frac{\delta}{\delta K^a_\mu(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta K^y_\mu(x)}\frac{\delta}{\delta W^\mu_y(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta W^\mu_y(x)}\frac{\delta}{\delta K^y_\mu(x)} \\ & + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta L_a(x)}\frac{\delta}{\delta C_a(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta C_a(x)}\frac{\delta}{\delta L_a(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta \nu_{L\alpha}(x)}\frac{\delta}{\delta n_\alpha(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta n_\alpha(x)}\frac{\delta}{\delta \nu_{L\alpha}(x)} \\ & + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta e_{L\alpha}(x)}\frac{\delta}{\delta l_\alpha(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta l_\alpha(x)}\frac{\delta}{\delta e_{L\alpha}(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta e_{R\alpha}(x)}\frac{\delta}{\delta p_\alpha(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta p_\alpha(x)}\frac{\delta}{\delta e_{R\alpha}(x)} \\ & + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta \overline{\nu}_{L\alpha}(x)}\frac{\delta}{\delta n'_\alpha(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta n'_\alpha(x)}\frac{\delta}{\delta \overline{\nu}_{L\alpha}(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta \overline{e}_{L\alpha}(x)}\frac{\delta}{\delta l'_\alpha(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta l'_\alpha(x)}\frac{\delta}{\delta \overline{e}_{L\alpha}(x)}\end{aligned}$$

$$+\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta\overline{e}_{R\alpha}(x)}\frac{\delta}{\delta p'_\alpha(x)}+\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta p'_\alpha(x)}\frac{\delta}{\delta\overline{e}_{R\alpha}(x)}\Big\}\,, \tag{4.27}$$

$$\Sigma_1(x)=\frac{\delta}{\delta\overline{C}_1(x)}-\partial_\mu\frac{\delta}{\delta K^1_\mu(x)}-g_1W_{y\mu}\frac{\delta}{\delta K^2_\mu(x)}-g_1W_{2\mu}\frac{\delta}{\delta K^y_\mu(x)}\,, \tag{4.28}$$

$$\Sigma_2(x)=\frac{\delta}{\delta\overline{C}_2(x)}-\partial_\mu\frac{\delta}{\delta K^2_\mu(x)}+g_1W_{y\mu}\frac{\delta}{\delta K^1_\mu(x)}+g_1W_{1\mu}\frac{\delta}{\delta K^y_\mu(x)}\,, \tag{4.29}$$

$$\Sigma_3(x)=\frac{\delta}{\delta\overline{C}_3(x)}-\partial_\mu\frac{\delta}{\delta K^3_\mu(x)}-g_1W_{y\mu}\frac{\delta}{\delta K^3_\mu(x)}-g_1W_{3\mu}\frac{\delta}{\delta K^y_\mu(x)}\,, \tag{4.30}$$

$$\Sigma_y(x)=\frac{\delta}{\delta\overline{C}_y(x)}-\partial_\mu\frac{\delta}{\delta K^y_\mu(x)}-gW_{y\mu}\frac{\delta}{\delta K^3_\mu(x)}-gW_{3\mu}\frac{\delta}{\delta K^y_\mu(x)}\,. \tag{4.31}$$

The meaning of the notation $A*B$ is the same as in the common use, namely

$$\begin{aligned}A*B=\int d^4x\Big\{&\frac{\delta A}{\delta K^a_\mu(x)}\frac{\delta B}{\delta W^\mu_a(x)}+\frac{\delta A}{\delta K^y_\mu(x)}\frac{\delta B}{\delta W^\mu_y(x)}+\frac{\delta A}{\delta L_a(x)}\frac{\delta B}{\delta C_a(x)}\\&+\frac{\delta A}{\delta\nu_{L\alpha}(x)}\frac{\delta B}{\delta n_\alpha(x)}+\frac{\delta A}{\delta e_{L\alpha}(x)}\frac{\delta B}{\delta l_\alpha(x)}+\frac{\delta A}{\delta e_{R\alpha}(x)}\frac{\delta B}{\delta p_\alpha(x)}\\&+\frac{\delta A}{\delta\overline{\nu}_{L\alpha}(x)}\frac{\delta B}{\delta n'_\alpha(x)}+\frac{\delta A}{\delta\overline{e}_{L\alpha}(x)}\frac{\delta B}{\delta l'_\alpha(x)}+\frac{\delta A}{\delta\overline{e}_{R\alpha}(x)}\frac{\delta B}{\delta p'_\alpha(x)}\Big\}\,.\end{aligned} \tag{4.32}$$

(4.24) – (4.26) are of course satisfied by the finite part and the pole part of $\overline{\Gamma}_1^{[0]}$. Thus the equations of the pole part $\overline{\Gamma}_{1,div}^{[0]}$ are

$$\frac{\delta\overline{\Gamma}_{1,div}^{[0]}}{\delta\lambda_a(x)}=0\,,\qquad\frac{\delta\overline{\Gamma}_{1,div}^{[0]}}{\delta\lambda_y(x)}=0\,, \tag{4.33}$$

$$\Lambda_{op}\overline{\Gamma}_{1,div}^{[0]}=0\,, \tag{4.34}$$

$$\Sigma_a(x)\overline{\Gamma}_{1,div}^{[0]}=0\,,\qquad\Sigma_y(x)\overline{\Gamma}_{1,div}^{[0]}=0\,. \tag{4.35}$$

Obviously, the same equations should be found for a $\mathrm{SU}_L(2)\times\mathrm{U}_Y(1)$ theory without the mass term if the same constraint conditions are chosen.

If $M=0$, then it is known from the renormalisability of the theory that $\overline{\Gamma}_{1,div}^{[0]}$ is a combination of the following terms

$$T_{GL}=g\frac{\partial\overline{\Gamma}_0^{[0]}}{\partial g}\,,\qquad T_{GY}=g_1\frac{\partial\overline{\Gamma}_0^{[0]}}{\partial g_1}\,,$$

$$T_{WL}=\int d^4x\Big\{W^\mu_a(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta W^\mu_a(x)}+L_a(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta L_a(x)}\Big\}\,,$$

$$T_{WY}=\int d^4x W^\mu_y(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta W^\mu_y(x)}\,,$$

$$T_{CK}=\int d^4x\Big\{\overline{C}_a(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta\overline{C}_a(x)}+C_a(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta C_a(x)}+K^a_\mu(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta K^a_\mu(x)}\Big\}\,,$$

$$T_{CKY}=\int d^4x\Big\{\overline{C}_y(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta\overline{C}_(x)}+C_y(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta C_y(x)}+K^y_\mu(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta K^y_\mu(x)}\Big\}\,,$$

$$T_{\nu L}=\int d^4x\Big\{\nu_{L\alpha}(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta\nu_{L\alpha}(x)}+\overline{\nu}_{L\alpha}(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta\overline{\nu}_{L\alpha}(x)}\Big\}\,,$$

$$T_{eL}=\int d^4x\Big\{e_{L\alpha}(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta e_{L\alpha}(x)}+\overline{e}_{L\alpha}(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta\overline{e}_{L\alpha}(x)}\Big\}\,,$$

$$T_{eR}=\int d^4x\Big\{e_{R\alpha}(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta e_{R\alpha}(x)}+\overline{e}_{R\alpha}(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta\overline{e}_{R\alpha}(x)}\Big\}\,,$$

$$T_{nn'}=\int d^4x\Big\{n_\alpha(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta n_\alpha(x)}+n'_\alpha(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta n'_\alpha(x)}\Big\}\,,$$

$$T_{ll'}=\int d^4x\Big\{l_\alpha(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta l_\alpha(x)}+l'_\alpha(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta l'_\alpha(x)}\Big\}\,,$$

$$T_{pp'}=\int d^4x\Big\{p_\alpha(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta p_\alpha(x)}+p'_\alpha(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta p'_\alpha(x)}\Big\}\,.$$

With these terms one can form five solutions of equations (4.33) − (4.35), which can be chosen as

$$T_{(1)}=T_{WL}-T_{GL}-T_{CK}\,, \tag{4.36}$$

$$T_{(2)}=T_{WY}-T_{GY}-T_{CKY}\,, \tag{4.37}$$

$$T_{(3)}=T_{CK}+T_{CKY}+T_{nn'}+T_{ll'}+T_{pp'}\,, \tag{4.38}$$

$$T_{(4)}=T_{\nu L}+T_{eL}-T_{nn'}-T_{ll'}\,, \tag{4.39}$$

$$T_{(5)}=T_{eR}-T_{pp'}\,. \tag{4.40}$$

Note that $T_{(3)}$ is $2\big(\overline{\Gamma}_0^{[0]}-I_{WL}-I_{WY}-I_\psi-I_{\psi W}\big)$. $T_{(1)}$ is a combination of I_{WL}, $T_{(3)}$ and $\int d^4xC_y(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta C_y(x)}$. $T_{(2)}$ is a combination of I_{WY} and $\int d^4xC_y(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta C_y(x)}$. The sum of $T_{(4)}$ and $T_{(5)}$ is $2\big(I_\psi+I_{\psi W}\big)$. $\int d^4xC_y(x)\frac{\delta\overline{\Gamma}_0^{[0]}}{\delta C_y(x)}$ and $T_{(5)}$ can be easily checked to satisfy (4.34) − (4.35). In addition to (4.36)−(4.40), a new term appearing in $\overline{\Gamma}_{1,\mathrm{div}}^{[0]}$ when $M\neq 0$ should includ M^2 as a factor and also satisfies (4.34) − (4.35). Only I_{WM} can be a candidate. Is such a term can really appear ? Imagine a limiting case that the matter fields and the $\mathrm{U}_Y(1)$ fields are absent. Thus the constraint conditions become Lorentz conditions and the above five solutions become two, namely, $\big(T_{WL}-T_{GL}\big)$ and T_{CK}. This combination of T_{WL} and T_{GL} are due to the restriction of the constraint condition containing $\partial^\mu W_{y\mu}$ and therefore should be decomposed into two independent terms when the $\mathrm{U}_Y(1)$ fields are absent. In fact, it is known that a SU(n) theory with massive gauge Bosons is renormalisability [1] and that when the matter fields are absent $\overline{\Gamma}^{[n]}_{n+1,div}$ of such a theory is a combination of three independent terms T_{WL}, T_{GL} and T_{CK}. It follows that for the present theory $\overline{\Gamma}^{[0]}_{1,div}$ does not cantain the mass term I_{MW} neither and can be expressed as

$$\overline{\Gamma}^{[0]}_{1,div} = \alpha_1^{(1)} T_{(1)} + \alpha_2^{(1)} T_{(2)} + \alpha_3^{(1)} T_{(3)} + \alpha_4^{(1)} T_{(4)} + \alpha_5^{(1)} T_{(5)} \,, \tag{4.41}$$

where, $\alpha_1^{(1)}, \cdots, \alpha_5^{(1)}$ are constants of order $(\hbar)^1$ and are divergent when the space-time dimension tends to 4.

In order to cancel the one loop divergence the counterterm of order $\hbar^1$ in the action should be chosen as

$$\delta I^{[1]}_{count} = -\overline{\Gamma}^{[0]}_{1,div} \,, \tag{4.42}$$

Since

$$\overline{I}^{[0]}_{eff} = \overline{\Gamma}^{[0]}_{0} \,, \tag{4.43}$$

it is known from (4.41) that the sum of $\overline{I}^{[0]}_{eff}$ and $\delta I^{[1]}_{count}$, to order of $\hbar^1$, can be written as

$$\begin{aligned} \overline{I}^{[1]}_{eff} \quad & [\psi, \overline{\psi}, W, C, \overline{C}, K, L, n, l, p, n', l', p', g, g_1] \\ & = \overline{I}^{[0]}_{\rm eff}[\psi^{[0]}, \overline{\psi}^{[0]}, W^{[0]}, C^{[0]}, \overline{C}^{[0]}, K^{[0]}, L^{[0]}, n^{[0]}, n^{'[0]}, \cdots, g^{[0]}, g_1^{[0]}] \,, \end{aligned} \tag{4.44}$$

where the bare fields and the bare parameters (to order $(\hbar)^1$) are defined as

$$W^{[0]}_{a\mu} = (Z_3^{[1]})^{1/2} W_{a\mu} = \big(1 - \alpha_1^{(1)}\big) W_{a\mu} \,, \quad L^{[0]}_a = (Z_3^{[1]})^{1/2} L_a \,, \tag{4.45}$$

$$W^{[0]}_{y\mu} = (Z_3^{'[1]})^{1/2} W_{y\mu} = \big(1 - \alpha_2^{(1)}\big) W_{y\mu} \,, \tag{4.46}$$

$$C^{[0]}_a = (\widetilde{Z}_3^{[1]})^{1/2} C_a = \big(1 - \alpha_3^{(1)} + \alpha_1^{(1)}\big) C_a \,, \tag{4.47}$$

$$\overline{C}^{[0]}_a = (\widetilde{Z}_3^{[1]})^{1/2} \overline{C}_a \,, \qquad K^{a[0]}_\mu = (\widetilde{Z}_3^{[1]})^{1/2} K^a_\mu \,, \tag{4.48}$$

$$C^{[0]}_y = (\widetilde{Z}_3^{'[1]})^{1/2} C_y = \big(1 - \alpha_3^{(1)} + \alpha_2^{(1)}\big) C_y \,, \tag{4.49}$$

$$\overline{C}^{[0]}_y = (\widetilde{Z}_3^{'[1]})^{1/2} \overline{C}_y \,, \qquad K^{y[0]}_\mu = (\widetilde{Z}_3^{[1]})^{1/2} K^y_\mu \,, \tag{4.50}$$

$$\nu^{[0]}_L = (Z^{[1]}_{\nu L})^{1/2} \nu_L = \big(1 - \alpha_4^{(1)}\big) \nu_L \,, \quad \overline{\nu}^{[0]}_L = (Z^{[1]}_{\nu L})^{1/2} \overline{\nu}_L \,, \tag{4.51}$$

$$e^{[0]}_L = (Z^{[1]}_{eL})^{1/2} e_L = (Z^{[1]}_{\nu L})^{1/2} e_L \,, \quad \overline{e}^{[0]}_L = (Z^{[1]}_{eL})^{1/2} \overline{e}_L \,, \tag{4.52}$$

$$e^{[0]}_R = (Z^{[1]}_{eR})^{1/2} e_R = \big(1 - \alpha_5^{(1)}\big) e_R \,, \quad \overline{e}^{[0]}_R = (Z^{[1]}_{eR})^{1/2} \overline{e}_R \,, \tag{4.53}$$

$$n^{[0]} = (Z^{[1]}_{(n)})^{1/2} n = \big(1 - \alpha_3^{(1)} + \alpha_4^{(1)}\big) n \,, \quad n^{'[0]} = (Z^{[1]}_{(n)})^{1/2} n' \,, \tag{4.54}$$

$$l^{[0]} = (Z^{[1]}_{(l)})^{1/2} l = (Z^{[1]}_{(n)})^{1/2} l \,, \quad l^{'[0]} = (Z^{[1]}_{(l)})^{1/2} l' \,, \tag{4.55}$$

$$p^{[0]} = (Z^{[1]}_{(p)})^{1/2} p = \big(1 - \alpha_3^{(1)} + \alpha_5^{(1)}\big) p \,, \quad p^{'[0]} = (Z^{[1]}_{(p)})^{1/2} p' \,, \tag{4.56}$$

$$g^{[0]} = Z_g^{[1]} g = (Z_3^{[1]})^{-1/2} g \,, \quad g_1^{[0]} = Z_g^{'[1]} g_1 = (Z_3^{'[1]})^{-1/2} g_1 \,. \tag{4.57}$$

Next, defined

$$\Phi_1^{[0]} = \partial^\mu W_{1\mu}^{[0]} + g_1^{[0]} W_{2\mu}^{[0]} W_y^{\mu[0]} \,,$$
$$\Phi_2^{[0]} = \partial^\mu W_{2\mu}^{[0]} - g_1^{[0]} W_{1\mu}^{[0]} W_y^{\mu[0]} \,,$$
$$\Phi_3^{[0]} = \partial^\mu W_{3\mu}^{[0]} + g_1^{[0]} W_{3\mu}^{[0]} W_y^{\mu[0]} \,,$$
$$\Phi_y^{[0]} = \partial^\mu W_{y\mu}^{[0]} + g^{[0]} W_{3\mu}^{[0]} W_y^{\mu[0]} \,.$$

From (4.45), (4.46) and (4.57) one has

$$g^{[0]} W_a^{\mu[0]} = g W_a^\mu \,, \qquad g_1^{[0]} W_y^{\mu[0]} = g_1 W_y^\mu \,,$$

and

$$\Phi_a^{[0]} = (Z_3^{[1]})^{1/2} \Phi_a \,, \quad \Phi_y^{[0]} = (Z_3^{'[1]})^{1/2} \Phi_y \,. \tag{4.58}$$

Thus by adding I_{WM} and the λ terms into $\overline{I}_{eff}^{[1]}$ and forming

$$I_{eff}^{[1]} = \overline{I}_{eff}^{[1]} + I_{WM} + \int d^4x \Big\{ \lambda_a(x)\Phi_a(x) + \lambda_y(x)\Phi_y(x) \Big\} \,, \tag{4.59}$$

one gets

$$\begin{aligned} &I_{eff}^{[1]}[\psi, \overline{\psi}, W, C, \overline{C}, \lambda, K, L, n, l, p, n', l', p', g, g_1, M] \\ &\quad = I_{eff}^{[0]}[\psi^{[0]}, \overline{\psi}^{[0]}, W^{[0]}, C^{[0]}, \overline{C}^{[0]}, \lambda^{[0]}, K^{[0]}, L^{[0]}, n^{[0]}, n^{'[0]}, \cdots, g^{[0]}, g_1^{[0]}, M^{[0]}] \,, \end{aligned} \tag{4.60}$$

where

$$M^{[0]} = (Z_3^{[1]})^{-1/2} M \,, \quad \lambda_a^{[0]} = (Z_3^{[1]})^{-1/2} \lambda_a \,, \quad \lambda_a^{[0]} = (Z_3^{'[1]})^{-1/2} \lambda_y \,. \tag{4.61}$$

Obviously, if the action $I_{eff}^{[1]}$ is used to replace $I_{eff}^{[0]}$ in (4.3) and define $\mathcal{Z}^{[1]}$, $\Gamma^{[1]}$ as well as

$$\overline{\Gamma}^{[1]} = \Gamma^{[1]} - I_{WM} - \int d^4x \Big\{ \lambda_a(x)\Phi_a(x) + \lambda_y(x)\Phi_y(x) + \mathcal{L}_{WM} \Big\} \,, \tag{4.62}$$

then one has

$$\begin{aligned} &\overline{\Gamma}^{[1]}[\psi, \overline{\psi}, W, C, \overline{C}, \lambda, K, L, n, l, p, n', l', p', g, g_1, M] \\ &\quad = \overline{\Gamma}^{[0]}[\psi^{[0]}, \overline{\psi}^{[0]}, W^{[0]}, C^{[0]}, \overline{C}^{[0]}, \lambda^{[0]}, K^{[0]}, L^{[0]}, n^{[0]}, n^{'[0]}, \cdots, g^{[0]}, g_1^{[0]}, M^{[0]}] \,. \end{aligned} \tag{4.63}$$

From this it is easy to check that, to order $\hbar^1$, $\overline{\Gamma}^{[1]}$ is finite. Moreover, by changing into bare fields and bare parameters the fields and parameters in (4.15)− (4.22) and then transforming them back into the renormalized fields and renormalized parameters according to (4.45)−(4.59), one can see that, under condition (4.23), $\overline{\Gamma}^{[1]}$ also satisfies

$$\Lambda_{op}\overline{\Gamma}^{[1]} = 0\,, \tag{4.64}$$

$$\frac{\delta\overline{\Gamma}^{[1]}}{\delta\lambda_a(x)} = 0\,, \qquad \frac{\delta\overline{\Gamma}^{[1]}}{\delta\lambda_y(x)} = 0\,, \tag{4.65}$$

$$\Sigma_a(x)\overline{\Gamma}^{[1]} = 0\,, \qquad \Sigma_y(x)\overline{\Gamma}^{[1]} = 0\,. \tag{4.66}$$

It is now clear that the renormalisability of the theory can be verified by the inductive method. The following is an outline of the proof. Assume that up to n loop the theory has been proved to be renormalisable by introducing the counterterm

$$I^{[n]}_{\rm count} = \sum_{l=1}^{n} \delta I^{[l]}_{\rm count},$$

where $\delta I^{[l]}_{count}$ is the counterterm of order $\hbar^l$ and has the form of (4.41),(4.42). Therefore the modified generating functional $\overline{\Gamma}^{[n]}$ for the regular vertex, defined by the action

$$I^{[n]}_{\rm eff} = I^{[0]}_{\rm eff} + I^{[n]}_{\rm count}$$

satisfied equations (4.64) − (4.66) (under (4.23)) and, to order $\hbar^n$, is finite. This also means that the fields or parameters in each of the following brackets have the same renormalization factor:

$$(W^{[0]}_{a\mu}, L_a), (C_a, \overline{C}_a, K^a_\mu), (C_y, \overline{C}_y, K^y_\mu), (\nu_L, \overline{\nu}_L, e_L, \overline{e}_L), (e_R, \overline{e}_R), (n, n', l, l'), (p, p'), (\lambda, M, g),$$

and that

$$Z'^{[n]}_g (Z'^{[n]}_3)^{1/2} = 1\,, \qquad Z^{[n]}_g (Z^{[n]}_3)^{1/2} = 1\,,$$

$$Z^{[n]}_3 \widetilde{Z}^{[n]}_3 = \widetilde{Z}'^{[n]}_3 \widetilde{Z}'^{[n]}_3 = Z^{[n]}_{\nu L} Z^{[n]}_{(n)} = Z^{[n]}_{eR} Z^{[n]}_{(p)}\,.$$

We have to proved that by using a counterterm of order $\hbar^{n+1}$ which also has the form of (4.41),(4.42), $\overline{\Gamma}^{[n+1]}$ can be make satisfy (4.64)−(4.66) and finite to order $\hbar^{n+1}$, where $\overline{\Gamma}^{[n+1]}$ is the modified generating functional for the regular vertex, determined by the action

$$I^{[n+1]}_{\rm eff} = I^{[n]}_{\rm eff} + \delta I^{[n+1]}_{\rm count}.$$

Denote by $\overline{\Gamma}^{[n]}_k$ the part of order $\hbar^k$ in $\overline{\Gamma}^{[n]}$. For $k \le n$, $\overline{\Gamma}^{[n]}_k$ is equal to $\overline{\Gamma}^{[k]}_k$, because it can not contain the contribution of a counterterm of order $\hbar^{k+1}$ or higher. Thus on expanding $\overline{\Gamma}^{[n]}$ to order $\hbar^{n+1}$ one has

$$\overline{\Gamma}^{[n]} = \sum_{k=0}^{n} \overline{\Gamma}^{[k]}_k + \overline{\Gamma}^{[n]}_{n+1} + \cdots\,.$$

Using this and extracting the terms of order $\hbar^{(n+1)}$ from the equations satisfied by $\overline{\Gamma}^{[n]}$, namely (4.64) – (4.66), one finds

$$\Lambda_{op}\overline{\Gamma}^{[n]}_{n+1} = 0\,, \tag{4.67}$$

$$\frac{\delta\overline{\Gamma}^{[n]}_{n+1}}{\delta\lambda_a(x)} = 0\,, \qquad \frac{\delta\overline{\Gamma}^{[n]}_{n+1}}{\delta\lambda_y(x)} = 0\,, \tag{4.68}$$

$$\Sigma_a(x)\overline{\Gamma}^{[n]}_{n+1} = 0\,, \qquad \Sigma_y(x)\overline{\Gamma}^{[n]}_{n+1} = 0\,, \tag{4.69}$$

Let $\overline{\Gamma}^{[n]}_{n+1,div}$ stand for the pole part of $\overline{\Gamma}^{[n]}_{n+1}$. By repeating the steps going from (4.33) to (4.41), one can arrive at

$$\overline{\Gamma}^{[n]}_{n+1,div} = \alpha_1^{(n+1)}T_{(1)} + \alpha_2^{(n+1)}T_{(2)} + \alpha_3^{(n+1)}T_{(3)} + \alpha_4^{(n+1)}T_{(4)} + \alpha_5^{(n+1)}T_{(5)}\,, \tag{4.70}$$

where $\alpha_1^{(n+1)}, \cdots, \alpha_5^{(n+1)}$ are constants of order $(\hbar)^{n+1}$. Therefore, in order to cancel the $n+1$ loop divergence the counterterm of order $\hbar^{n+1}$ should be chosen as

$$\delta I^{[n+1]}_{count} = -\overline{\Gamma}^{[n]}_{n+1,div}[\psi, \overline{\psi}, W, C, \overline{C}]\,. \tag{4.71}$$

Adding this counterterm, the mass term and the λ terms to $\overline{I}^{[n]}_{\rm eff}$, one can express the effective action of order $\hbar^{n+1}$ as

$$\begin{aligned} & I^{[n+1]}_{\rm eff}[\psi, \overline{\psi}, W, C, \overline{C}, \lambda, K, L, n, l, p, n', l', p', g, g_1, M] \\ & \quad = I^{[0]}_{\rm eff}[\psi^{[0]}, \overline{\psi}^{[0]}, W^{[0]}, C^{[0]}, \overline{C}^{[0]}, \lambda^{[0]}, K^{[0]}, L^{[0]}, n^{[0]}, n'^{[0]}, \cdots, g^{[0]}, g_1^{[0]}, M^{[0]}]\,, \end{aligned} \tag{4.72}$$

where the bare fields and the bare parameters (to order $(\hbar)^{n+1}$) are defined as

$$W^{[0]}_{a\mu} = (Z_3^{[n+1]})^{1/2}W_{a\mu} = \big((Z_3^{[n]})^{1/2} - \alpha_1^{(n+1)}\big)W_{a\mu}\,, \quad L_a^{[0]} = (Z_3^{[n+1]})^{1/2}L_a\,, \tag{4.73}$$

$$W^{[0]}_{y\mu} = (Z_3'^{[n+1]})^{1/2}W_{y\mu} = \big((Z_3'^{[n]})^{1/2} - \alpha_2^{(n+1)}\big)W_{y\mu}\,, \tag{4.74}$$

$$C_a^{[0]} = (\widetilde{Z}_3^{[n+1]})^{1/2}C_a = \big((\widetilde{Z}_3^{[n]})^{1/2} + (-\alpha_3^{(n+1)} + \alpha_1^{(n+1)})\big)C_a\,, \tag{4.75}$$

$$\overline{C}_a^{[0]} = (\widetilde{Z}_3^{[n+1]})^{1/2}\overline{C}_a\,, \qquad K_\mu^{a[0]} = (\widetilde{Z}_3^{[n+1]})^{1/2}K_\mu^a\,, \tag{4.76}$$

$$C_y^{[0]} = (\widetilde{Z}_3'^{[n+1]})^{1/2}C_y = \big((\widetilde{Z}_3'^{[n]})^{1/2} + (-\alpha_3^{(n+1)} + \alpha_2^{(n+1)})\big)C_y\,, \tag{4.77}$$

$$\overline{C}_y^{[0]} = (\widetilde{Z}_3'^{[n+1]})^{1/2}\overline{C}_y\,, \qquad K_\mu^{y[0]} = (\widetilde{Z}_3^{[n+1]})^{1/2}K_\mu^y\,, \tag{4.78}$$

$$\nu_L^{[0]} = (Z_{\nu L}^{[n+1]})^{1/2}\nu_L = \big((Z_{\nu L}^{[n]})^{1/2} - \alpha_4^{(n+1)}\big)\nu_L\,, \quad \overline{\nu}_L^{[0]} = (Z_{\nu L}^{[n+1]})^{1/2}\overline{\nu}_L\,, \tag{4.79}$$

$$e_L^{[0]} = (Z_{eL}^{[n+1]})^{1/2}e_L = (Z_{\nu L}^{[n+1]})^{1/2}e_L\,, \quad \overline{e}_L^{[0]} = (Z_{eL}^{[n+1]})^{1/2}\overline{e}_L\,, \tag{4.80}$$

$$e_R^{[0]} = (Z_{eR}^{[n+1]})^{1/2}e_R = \big((Z_{eR}^{[n]})^{1/2} - \alpha_5^{(n+1)}\big)e_R\,, \quad \overline{e}_R^{[0]} = (Z_{eR}^{[n+1]})^{1/2}\overline{e}_R\,, \tag{4.81}$$

$$n^{[0]} = (Z^{[n+1]}_{(n)})^{1/2} n = \big((Z^{[n]}_{(n)})^{1/2} + (-\alpha_3^{(n+1)} + \alpha_4^{(n+1)})\big) n \,, \quad n'^{[0]} = (Z^{[n+1]}_{(n)})^{1/2} n' \,, \tag{4.82}$$

$$l^{[0]} = (Z^{[n+1]}_{(l)})^{1/2} l = (Z^{[n+1]}_{(n)})^{1/2} l \,, \quad l'^{[0]} = (Z^{[n+1]}_{(l)})^{1/2} l' \,, \tag{4.83}$$

$$p^{[0]} = (Z^{[n+1]}_{(p)})^{1/2} p = \big((Z^{[n]}_{(p)})^{1/2} - \alpha_3^{(n+1)} + \alpha_5^{(n+1)}\big) p \,, \quad p'^{[0]} = (Z^{[n+1]}_{(p)})^{1/2} p' \,, \tag{4.84}$$

$$g^{[0]} = Z^{[n+1]}_g g = (Z^{[n+1]}_3)^{-1/2} g \,, \quad g_1^{[0]} = Z'^{[n+1]}_g g_1 = (Z'^{[n+1]}_3)^{-1/2} g_1 \,, \tag{4.85}$$

$$g^{[0]} = Z^{[n+1]}_g g = (Z^{[n+1]}_3)^{-1/2} g \,, \quad g_1^{[0]} = Z'^{[n+1]}_g g_1 = (Z'^{[n+1]}_3)^{-1/2} g_1 \,, \tag{4.86}$$

$$M^{[0]} = Z^{[n+1]}_M M = (Z^{[n+1]}_3)^{-1/2} M \,, \tag{4.87}$$

and $\lambda_a^{[0]}, \lambda_y^{[0]}$ are

$$\lambda_a^{[0]} = (Z^{[n+1]}_3)^{-1/2} \lambda_a \,, \quad \lambda_y^{[0]} = (Z'^{[n+1]}_3)^{-1/2} \lambda_y \,. \tag{4.88}$$

Therefore, in terms of such bare fields and bare parameters, $\overline{\Gamma}^{[n+1]}$ can be expressed as

$$\begin{aligned} \overline{\Gamma}^{[n+1]} \quad & [W, C, \overline{C}, \psi, \overline{\psi}, K, L, n, l, p, n', l', p', g, g_1, M] \\ & = \overline{\Gamma}^{[0]} [W^{[0]}, C^{[0]}, \overline{C}^{[0]}, \psi^{[0]}, \overline{\psi}^{[0]}, K^{[0]}, L^{[0]}, n^{[0]}, n'^{[0]}, \cdots, g^{[0]}, g_1^{[0]}, M^{[0]}] \,. \end{aligned} \tag{4.89}$$

From this one can conclude that $\overline{\Gamma}^{[n+1]}$, under (4.23), satisfies (4.64)$-$(4.66) and is finite to order $\hbar^{n+1}$. Since the theory can be renormalized to one loop the renormalisability has been proven.

V. Concluding Remarks

By taking into account the original constraint conditions and the additional condition we have carried out the quantization of the $\mathrm{SU}_L(2) \times \mathrm{U}_Y(1)$ electroweak theory with the W Z mass term and construct the ghost action in a way similar to that used for the massive SU(n) theory [1]. We have also shown that when the $\delta-$ functions appearing in the path integral of the Green functions and representing the constraint conditions are rewritten as Fourier integrals with Lagrange multipliers λ_a and λ_y, the total effective action consisting of the Lagrange multipliers, ghost fields and the original fields is BRST invariant. Furthermore, by comparing with the massless theory and with the massive SU(n) theory we have found the general form of the divergent part of the generating functional for the regular vertex functions and proven the renormalisability of the theory. It has also been clarified that the renormalisability of the theory with the W Z mass term is ensured by the renormalisability of the massless theory and the massive SU(n) theory.

If the harmlessness of the W Z mass term had been proven at the begining of 1960s, the $SU_L(2) \times U_Y(1)$ electroweak theory without the Higgs mechanism would have been deeply studied and tested. Today, the standard model of the electroweak theory has achieved great successes and the whereabouts of the Higgs Bosons is still unknown. It is therefore reasonable to ask if such successes really depends on the Higgs mechanism and to pay attention to the theory without the Higgs mechanism.

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